Projective Geometry for Computer Vision

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3D Computer Vision

Classical Problem:
Given a collection of 2D images, build a model of the 3D world.

Example Applications:
- virtual/immersive environments
- robotics & autonomous vehicles
- minimally invasive surgery
Outline

1. Projective Geometry Overview
2. Minimal Projective Parameters
3. Projective Parameter Estimation
4. Motion Boundary Detection
5. Conclusion
Image Formation

3D scene → imaging → 2D images
Computer Vision

3D scene model  ➔  data  ➔  2D images

analysis  ➔  measurement

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May 12, 2004

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Camera Geometry: Single View

pinhole model of perspective projection

unknown depth at each point

unknown internal camera parameters

\[
x = \frac{X}{Z} \quad y = \frac{Y}{Z}
\]

\[
\begin{bmatrix}
x \\
y
\end{bmatrix} \rightarrow \begin{bmatrix}
f_x & s & \cdot \\
1 & f_y & \cdot 
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix} + \begin{bmatrix}
c_x \\
c_y
\end{bmatrix}
\]
Camera Geometry: Multiple Views

unknown rotations and translations

\[
\begin{bmatrix}
X \\
Y \\
Z \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
X \\
Y \\
Z \\
\end{bmatrix}
+ T
\]
Measured Data: Image Points and Lines

geometric constraint: optical rays intersect in 3D

projective geometry: express constraint in terms of measured 2D image features
Projective Camera Model

- linear model of image formation
- depth-independent expression for optical ray intersections
- multilinear relations among point and line matches
Bilinear Constraints

\[ X = \lambda_i x_i \]

\[ \lambda_j x_j = \lambda_i R_{ij} x_i + T_{ij} \]

\[ x_j^T [T_{ij}] x R_{ij} x_i = 0 \]

\[ x_i \rightarrow A_i^{-1} x_i \]

\[ x_j \rightarrow A_j^{-1} x_j \]

\[ x_j^T A_{ij}^{-T} [T_{ij}] R_{ij} A_{ij}^{-1} x_i = 0 \]

\[ x_j^T F_{ij} x_i = 0 \]

fundamental matrix
Fundamental Matrix

Maps a point in one image to a line in the other image that contains its match.

Given matching points in two views, predict the matching point in a third image.
Projective Models in Practice

• View synthesis and interpolation: point transfer function for dense point correspondences

• Self-calibration: automatic recovery of internal camera parameters from fundamental matrices

• Bundle adjustment initialization: initial rotation and translation for nonlinear Euclidean optimization
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Practical Problem

- Few point matches between some views.
- Unstable for estimating geometric relationships.
Geometric Consistency

Pairwise geometric relations may be inconsistent.
Goals

• Impose algebraic geometric constraints on stationary points seen in arbitrarily many views.

• Avoid estimating too many parameters: depths, rotations, translations
Geometric Dependencies

- Pairwise projective geometric relations are interdependent.

- Approach: define projective dependencies and restrict solutions to be globally consistent.
Projective Bilinear Parameters

\[ x_j^T F_{ij} x_i = 0 \]

\[ F_{ij} = A_j^{-T} \left[ T_{ij} \right]_x R_{ij} A_i^{-1} \]
Projective Bilinear Parameters

\[ x_j^T F_{ij} x_i = 0 \]

epipoles
\[ e_{ij} \quad e_{ji} \]
epipolar collineation
\[ h_{ij} \]

imaged 3D translation & rotation

\[ F_{ij} \approx [e_{ji}]_x [p_j \quad q_j] h_{ij} \begin{bmatrix} q_i^T \\ -p_i^T \end{bmatrix} [e_{ij}]_x \]

(Csurka, et.al., 1997)
Projective Parameters

• provide a complete projective model of camera configuration

But...

• set of all pairwise parameters are still redundant

• not all images have sufficient overlap
Trifocal Dependencies

• derive dependencies among three fundamental matrices

• correctly models degrees of freedom in camera configuration

• geometrically consistent parameterized model of view triplets
Trifocal Dependencies

- derive dependencies among three fundamental matrices
- correctly models degrees of freedom in camera configuration
- geometrically consistent parameterized model of view triplets

trifocal lines available from two fundamental matrices
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Recovering Camera Geometry

view i  view k  view j

few correspondences

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Linear Initialization
8-point Algorithm
(Hartley, 1995)

Minimize \[ \sum_{i,j} \left( x_i^T F_{ij} x_i \right) \]
over all matching point pairs.

Rewrite bilinear constraints as
\[ \begin{bmatrix} x_i x_j & y_i x_j & x_i y_j & y_i y_j & y_j & x_j & y_j & 1 \end{bmatrix} f_{ij} = 0 \]
where
\[ f_{ij} = [f_{11} f_{12} f_{13} f_{21} f_{22} f_{23} f_{31} f_{32} f_{33}]^T \]
and solve linear system
\[ A f_{ij} = 0 \]
Projection to Parameter Space

Map linear estimate of fundamental matrix to projective parameter space:

\[ F_{ij} \rightarrow p_{ij} = \{e_{ij}, e_{ji}, h_{ij}\} \rightarrow p_{4}^{ij} = \{\gamma_i, \gamma_j, h_{ij}\} \]

- parameterization requires choice of projective basis
- basis affects shape of error surface for nonlinear optimization
Geometric Objective Function

point-to-epipolar-line distance ~ image reprojection error

weighted residual of bilinear constraint

\[ E(x_i, x_j; p_{ij}) = w_{ij} \ x_j^T F_{p_{ij}} x_i \]

\[ w_{ij} = \frac{1}{(F_{ij} x_i)_1^2 + (F_{ij} x_i)_2^2} + \frac{1}{(F_{ij}^T x_j)_1^2 + (F_{ij}^T x_j)_2^2} \]
Error Surface Depends on Basis

canonical basis

geometrically defined basis

\( \gamma(i,j) \)
Nonlinear Trifocal Estimation

1. Initialize epipolar geometry
   - 8-point algorithm: linear solution to fundamental matrix for all view pairs
   - extract epipoles and epipolar collineations
2. 7D nonlinear minimization: bifocal parameters for view pairs \((i,k) (j,k)\)
3. Trifocally constrained estimation for view pair \((i,j)\)
   - compute trifocal lines
   - project parameters to trifocally constrained space
   - 4D nonlinear minimization for bifocal parameters
Convergence

- Convergence plots for different algorithms:
  - Ground Truth
  - 8-point Algorithm
  - 7-Parameter Search
  - Trifocal Projection
  - 4-Parameter Search

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knossos sequence

view i

view k

view j

few correspondences

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Ground Truth

Results

8-Point Algorithm

7-Parameter Algorithm

4-Parameter Algorithm
Summary

• Imposing projective constraints on camera geometry corrects the estimation of epipolar geometry

• Resulting camera configuration for multiple cameras is globally consistent
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Camera and Scene Motion
Combining Intensity and Geometry

trifocal tensor

projective linear form relating a point-line-line

(Spetsakis & Aloimonos, 1990; Shashua, 1994)

\[ T(x_i, l_j, l_k) = 0 \]
Tensor Brightness Constraint
(Shashua & Hannah, 1995; Shashua & Stein, 1997)

\[ u I_x + v I_y + I_t = 0 \]

\[ u = x - x_0 \quad v = y - y_0 \]

\[ ax + by + c = 0 \]

\[ (a,b,c)^T \cong \begin{bmatrix} I_x \\ I_y \\ I_t - x_0 I_x - y_0 I_y \end{bmatrix} \]

- Horn-Schunk brightness constraint is linear in point coordinates
- Defines line in each image containing matching point
- Spatiotemporal gradient at every pixel provides test of rigid motion
Motion Boundary Detection

• Partition image into windows and solve for trifocal tensor coefficients.

• Only regions with rigid 3D motion have a good fit.

• Sum residual error of tensor solution.

• High residuals indicate regions that cross a motion boundary.
Multiple Frame Flow

- Track points over many frames.
- Multi-frame tracks fall into separable classes.
- Robustly fit tracks to linear approximation of instantaneous planar motion.

\[ x(t) = x_0 + t [Ax_0 + b] \]
Detecting Independent Motions

Residual error of estimated motion model on all point tracks
Complexity of Motion Model
Conclusions

When possible, use domain and task knowledge to choose model:

• What type of information is needed
• What aspects of the imaging conditions are known or controlled
• What types of uncertainty can be modeled and compensated for
Future Needs

Role of learning in motion analysis:

• Supervised learning of geometric motion classes
• Data-driven model selection by flow classification
• Robust estimation of appropriate motion model
• Adaptive, time-varying estimation